

ON A THEORY OF QUATERNIONS

By

William Rowan Hamilton

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On a Theory of Quaternions. By Sir WILLIAM R. HAMILTON, M.R.I.A.

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It has been shown, by Mr. Warren and others, that the results obtained by the ordinary processes of algebra, involving the imaginary symbol $\sqrt{-1}$, admit of real interpretations, such as those which relate to compositions of linear motions and rotations in one plane. Sir W. Hamilton has adopted a system of three such imaginary symbols, i , j , k , and assumes or defines that they satisfy the nine equations

$$i^2 = j^2 = k^2 = -1, \quad ij = k = -ji, \quad jk = i = -kj, \quad ki = j = -ik,$$

which however are not purely arbitrary, and for the adoption of which the paper assigns reasons. He then combines these symbols in a *quaternion*, or imaginary quadrinomial, of the form

$$Q = w + ix + jy + kz,$$

in which w , x , y , z , are four real quantities; and states that he has established rules for algebraical operations on such expressions, and has assigned geometrical interpretations corresponding; so as to form a sort of *Calculus of Quaternions*, which serves as an instrument to prove old theorems, and to discover new ones, in the geometry of three dimensions, and especially respecting the composition of motions of translation and rotation in space.